Critical speed is one of several important design criteria for shafts. Commonly Dunkerley’s Equation and/or Rayleigh’s Methods are employed to accomplish the calculation. Both methods rely on a priori determination of shaft deflection at several locations. For uniform shafts with simple loading the calculations are straight-forward. For any number of practical reasons, however, shafts are neither uniform nor simply-loaded. Critical speed determination for these “practical” shafts can become tedious. Teaching undergraduate students to conduct such calculations is challenging at least.

This presentation shows a method for teaching undergraduate students to accomplish the calculations. We have found that students, overwhelmingly, can learn and use the method with accuracy and understanding. The method requires that the student can write a moment equation using discontinuity terms, take a partial derivative, and use an equation-solver such as TKSolver™ or EES©. Beginning with the internal energy due to bending and Castigliano’s second theorem for deflection: \[ U = \int_{0}^{L} \frac{M^2}{2EI} \, dx, \quad \text{and} \quad \delta = \frac{\partial U}{\partial Q} = \int_{0}^{L} \frac{M}{EI(x)} \frac{\partial M}{\partial Q} \, dx, \] where \( M \) is the moment along the length of the beam as a function of \( x \), \( E \) is Young’s modulus, and \( I \) is the area moment of inertia (which for a stepped shaft is also a function of \( x \)).

**Keywords:** Shaft design, non-uniform diameter, deflection, equation-solving software, Castigliano’s Theorem.

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