Abstract - The study of structural analysis includes many critical details that may seem abstract to students. The concepts become more meaningful once they can be applied to larger problems. This problem summarizes such an exercise. The ACI 318 moment coefficients [1] for simplified analysis are studied in the context of topics important to an undergraduate studying structural analysis. Specifically, this exercise requires students to draw on knowledge of influence lines, indeterminate analysis, and software-based analysis to study the published moment coefficients.

Keywords: ACI 318, structural analysis, influence lines.

Code Provisions and Set-Up

ACI 318 section 8.3.3 summarizes code provisions for moment coefficients that simplify the analysis of continuous beams and one way slabs. They consider both the transient nature of live load and the various boundary conditions common in concrete structures. These code provisions require two or more continuous spans, spans of equal length or having the larger of two adjacent spans not being greater than the shorter by more than 20 percent, loads being uniformly distributed, unfactored live load not exceeding three times unfactored dead load, and the members being prismatic. All provisions must be met when using the moment coefficients; however, it is not apparent from the code language whether further provisions must also be met to ensure that the moment coefficients apply when analyzing a structure.

The moment coefficients examined in this study are as followed:

- : maximum positive moment in the end span for a discontinuous end unrestrained
- : maximum positive moment in the end span for a discontinuous end integral with support
- : maximum positive moment in an interior span

This study produces results from a detailed analysis which confirms the moment coefficients and clarifies the scope of the method.

The model created using commercially available structural analysis software was a four span beam of 25 foot segments with 20 foot long support beams attached to each joint in both directions. A uniform dead load of 1 kip per linear foot was added with a factor of 1.2, and a uniform live load of 3 kip per linear foot with a factor of 1.6 was applied to segment of the beam. These criteria met the ACI 318 section 8.3.3 code provisions for the moment coefficients. A conceptual graphic of the computer model is shown in Figure 1.

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One item not categorized in the code provisions are the load positions on the beam. The moment coefficient must properly consider the transient nature of live load, and represent the worst-case scenario. The dead load is placed over the entire beam, but the live load could be placed over any of the four spans. It might be obvious to think that a worst case scenario would be placing a live load across the whole beam, but that is not the case. When examining a two span beam with the point of interest being the midspan of the first span, it is found that a uniform load across only the first span would create a worst case scenario. This is due to the uniform load in the second span resisting an upwards force created by the deflection of the first span. To better clarify this point, Figure 2 illustrates a 2 span beam with live load placements and their moment diagrams.
Introduction of Influence Lines

Instead of determining worst case scenarios by figuring out every possible configuration and testing them, in the four span continuous beam, it is easiest to determine where the live load should be placed using a structural analysis tool called influence lines. An influence line allows one to view the variation of shear, moment, reactions, or deflections of a single point as a force moves along an entire member. Figure 3 displays moment influence lines of the four span beam with interest points at the midspan, F, of the end span (relevant for the 1/11 and 1/14 coefficients), and the midspan, G, of the interior span (relevant for the 1/16 coefficient).
Examination and Comparison to Basic Structural Analysis

The end span moment coefficients $w_i l_i^2 / 11$ and $w_i l_i^2 / 14$ have only one thing differentiating the two, one has unrestrained ends, and the other has a supported end. To understand why this is, it is easier to look at a more basic example in Figure 4.

In condition A, the beam has a pinned support on the left and a roller on the right with a uniform load across it representing an unrestrained end. In condition B, the only difference is a fixed support on the left representing a restrained or integral supported end. When superimposing both condition A and B’s moment diagrams onto the same graph, as in Figure 5, it is apparent that the maximum moment for condition B is much smaller than for condition A.
The significant difference is solely dependent on the fixed end. The support’s ability to transfer moment reduces the positive moment the span takes. This is reflected in the moment coefficients, by having the discontinuous end integral with support produce a smaller positive moment than the unrestrained when calculated.

The interior span moment coefficient \( w_\alpha \frac{l^2}{16} \) is fairly unique compared to more commonly known maximum bending moment equations \( w\frac{l^2}{8} \) for a uniformly loaded simply supported or pinned-pinned span, and \( w\frac{l^2}{24} \) for a uniformly loaded fixed-fixed span. The interior span can be represented by condition D, with the other two represented by conditions C and E shown in Figure 6.

![Figure 6. Moment Equations](image)

When superimposing the three conditions’ moment diagrams on a graph, as seen in Figure 7, it is apparent that the maximum positive moment for the interior span falls in between that of the pinned-pinned span, and the fixed-fixed span.
Figure 7. Moment Diagram of Spans C, D and E

Figure 7 demonstrates the flexural stiffness afforded by the adjacent spans. The stiffness of the adjacent spans is not infinite, as would be the case if both end nodes were fixed (Case E). But the end nodes also do not have zero stiffness (Case C). Instead, the stiffness provided by the adjacent spans brings the behavior a point somewhere between the extremes of Case C and Case E.

Study of Torsional Stiffness

It is not apparent, from the code language, whether torsional stiffness in the out-of-plane support beams affects the analysis results and choice of moment coefficient. This was studied through a simple parametric model using commercially available structural analysis software. A modified setup was created with the 20 foot long support beams being unrestrained for torsion in the Z axis. Figure 8 shows the modified setup.
The analysis produced a maximum positive moment value which was considerably larger than the span with just ends unrestrained. The unrestrained torsional support moment diagram can be viewed at the bottom of Figure 8, and the restrained torsional support moment diagram can be viewed in Figure 10. This confirmed that the moment coefficients indeed assume that the support beams provide torsional stiffness. A 2-dimensional visualization of the beam with added torsional stiffness can be seen in Figure 9. The nodes include the addition of torsional springs representing the added stiffness that absorbs part of the moment from the continuous beam. The moment coefficients in ACI 318 section 8.3.3 must only be used when torsional restraint from out-of-plane beams exists.
In this study of structural analysis, the end results were much more than the confirmation of a few moment coefficients. It involved realizing the value of each individual step to learning a complicated technique to solving a challenging problem. This study showed how a basic example could demonstrate the difference in moment coefficients, or provides a step for understanding live load placement. That step was then implemented into using a moment influence line to find the correct loading positions for points of interest in the four span continuous beam. Finally, a simple parametric study was used in a representative model to understand the impact of torsional stiffness from the out-of-plane girders on the flexural analysis results. This project pulled together many important concepts for a student of structural analysis.

REFERENCES

[1] American Concrete Institute Committee 318, BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE (ACI 318-95), FARMINGTON HILLS, MI 48333, 1995, pages 83-84.

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Thomas is a third year Civil Engineering undergraduate student at the University of Hartford currently working on an honors degree. While completing the Structural Analysis course for honors credit, a parametric model of six moment coefficients from the ACI 318 section 8.3.3 code provisions, three which are presented in this paper, were studied and confirmed using commercially available structural analysis software. He is also a national and University of Hartford student chapter member of ASCE and has aspirations of obtaining a summer internship.

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