Library of MATLAB Modules for Solving and Visualizing Electromagnetics Problems

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Abstract – Students that possess a firm grasp of the fundamentals involved in solving electrostatics and electrodynamics problems are well equipped to proceed in the fields of electrical and biomedical engineering. Relevant concepts hold true for more advanced course work and can provide a basis for research into a variety of areas. This paper introduces a library of simple MATLAB® based modules employed to facilitate both the instruction and understanding of electromagnetics concepts for educators and students. Topics covered include: the electric field, surface and volume charges, material constitutive parameters, reflection and transmission in free space and between different media, wave polarization, critical angle, power transfer, Zenneck surface waves, plasmons, trapped surface waves, and surface impedance. These modules may augment undergraduate and graduate classes in electrical and computer engineering, biomedical engineering, and introductory physics and many have been created to complement Ulaby’s Fundamentals of Applied Electromagnetics text.

Keywords: Electrostatics, electrodynamics, engineering education, electrical engineering

INTRODUCTION

When first encountering the numerous topics involved in electromagnetics, the task of mastering this wonderful field of study can be daunting. From vector calculus and coordinate transformations to solve Maxwell’s equations on simple geometries to the description of electric boundary conditions that govern the electric field between two media, there is a significant amount of material to internalize and understand before progressing on to more advanced topics and problems. Many of the nuances and interrelations between quantities can be lost in the deluge of new concepts and terminology that abound.

While a student may feel overwhelmed by the breadth of material to be studied, an educator in the field may feel a similar pressure to thoroughly cover all the material in a way that penetrates the minds of each student in the class. For some students, the standard in-class tools and lectures are insufficient to fully adopt concepts and these students often require more time and energy from the instructor to overcome any gaps in understanding.

The solution and visualization tools described in this paper have been designed with both student and instructor in mind. Written in basic MATLAB with no additional toolboxes required, these modules cover much of the material involved in an introductory electromagnetics class and provide a means to visualize concepts without focusing solely on the underlying mathematics. The plane wave is utilized as an intuitive approach in solving Maxwell’s equations and a starting point for an angular-spectrum method, also called the spectral method or the k-space method, which is a powerful numerical method in solving Maxwell’s equations [1]-[6]. There are a number of canonic problems, including reflection and refraction, which are solved using this approach. In this way, underlying

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ideas can be introduced and interactively explored in an approachable way while providing the instructor with additional flexibility to reach all students in a methodical and detailed manner.

**Electrostatics Module Topics**

The topics covered by the current library of electrostatics modules include capacitance of single and multiple arbitrary conducting bodies, electrostatic behavior of conducting materials, and electrostatics of dielectrics. Variation of the constitutive parameters of materials is supported along with current flow in conductive media. Solution for the electrostatic charge is found via the Method of Moments (MoM) or Boundary Element Method (BEM) [7]-[10]. This powerful numerical technique offers a simple and fast method to obtain accurate solutions and can be augmented by the collocation method as described in [11]-[13] and adaptive mesh refinement. However, use of the library is not dependent on knowledge of the underlying methods. The numerical process, the iterative method used to obtain solutions and the generation of model meshes via DISTMESH [14]-[15] is masked by an intuitive and interactive graphical user interface (GUI). This allows both the instructor and student to concentrate solely on the concepts of electrostatics and offers a dynamic mechanism of exploration. Topics are described in detail below.

**Self Capacitance**

Self capacitance is defined for an isolated conductor as the amount of electrical charge that must be added to raise the electric potential of the conductor by one volt. Analytical solutions for the self capacitance of many simple shapes exist and can provide easy validation for numerical solutions. However, when dealing with complicated shapes, closed form solutions are intractable and approximations via numerical methods are required for a solution. The modules described in this paper can provide a bridge between the simple analytical models and more complicated geometries.

An example of module operation is shown in Fig. 1. At left, the GUI and geometry initialization is shown. In this case a cylinder is defined, but other objects, including bricks, spheres, and plates are supported. Parameters of object position and size may be varied along with the number of triangular elements used to represent the object. Planes of observation can be defined to facilitate plotting of electric fields and lines of equipotential. The results are shown at right in Fig. 1 where the surface charge along the cylinder is plotted. Lines of equipotential radiate outward from the object and the magnitude of the electric field is proportional to the size of the arrows that are shown to be decreasing with distance.

![Fig.1. Evaluation of self-capacitance of a cylinder and calculation of surrounding electrostatic field.](image)

**Mutual Capacitance**

Traditionally, mutual capacitance is introduced by exploring the physics that define a parallel plate capacitor. The equation governing capacitance in this case is proportional to capacitance area, $A$, distance separating the plates, $d$, and the permittivity of the material between the plates, and for an air filled capacitor, is given as:
\[ C = \frac{\varepsilon_0 A}{d} \]  

Equation (1) is valid when the distance separating the plates is small in relation to the plate area. However, when this distance becomes large, Eq. (1) no longer provides an accurate solution due to field fringing around the plate edges. When this happens, more advanced expressions are required to correctly represent the capacitance value. Numerical solutions such as the one depicted in Fig. 2 are also an option.

On the left side of Fig. 2, the module GUI is shown and a parallel plate capacitor is defined. The capacitor may be composed of arbitrary shapes (spheres, plates, bricks, cylinders, etc.) and the sizes and positions of either conducting body may be changed. Additionally, conducting and dielectric materials may be inserted into the space between the conductors and the properties of this material may be changed as required, seen in Fig. 3. A plane bisecting the capacitor center is given and used by the module to project lines of equipotential and the electric field, as shown in the figure at right. Surface charge along each conductor is also plotted and improvements in accuracy may be realized via adaptive mesh refinement. Field fringing can be interactively explored by rotating the figure and viewing results in three dimensions. The effect is exacerbated when plate separation is large.

![Fig. 2. Mutual capacitance of a parallel plate capacitor with plots of surface charge and electric field. Note fringing effect.](image)

![Fig. 3. Capacitance of a parallel disk capacitor with an internal conducting sphere.](image)
Figure 3 above displays a conducting sphere inserted a capacitor composed of two identical flat disks. The module allows for modification of the internal and external object types as well as the positions, orientations and sizes of all modeled items. Of particular interest is how the equipotential is affected by the presence of this additional body and how surface charge is distributed along the sphere. These phenomena can be extended to a charged or uncharged dielectric object placed between two conducting objects and material variations provide interesting differences observed in the field internal to the conductors.

Fig. 4. Evaluation of self-capacitance of a cylinder and calculation of surrounding electrostatic field.

**Conductor or Dielectric in External Field**

On the left of Fig. 4, a conductive sphere is shown suspended in an external uniform electric field. The effects on the field as a result of this object can be quickly and easily explored through the use of the module and are shown at the right of Fig. 4. The object type may be varied between a plate, sphere, brick or cylinder and other properties, including object size, position and orientation, may be modified as required. The magnitude and direction of the external field may be changed to examine how field intensity and orientation produce different resultant patterns.

**Extension to Human Body Modeling and Research Applications**

As mentioned earlier, analytical solutions to many simple geometries exist in the literature and provide a number of canonical problems to explore. These problems can provide vital verification and validation to modeling activities like the ones described above. However, when exploring complex geometries like the human body, no previous analytical solutions exist and experimental data can be sparse. These applications require robust and accurate modeling tools that are easily understandable and provide fast solutions.

The modules described above can be extended to applications involving the human body. Using a collection of body meshes obtained via a 3D whole body scanner, body self capacitance and the effects of a body on an external field may be calculated. A body could also be inserted into a capacitive field. Several of these concepts are shown in Fig 5 below. Examining the behavior of various static quantities as a function of body position, size and distance to the ground can provide illumination into how body composition can effect results.

**Undergraduate Lab Experiments.**

The concept of human body self capacitance has been examined experimentally to be approximately 50-250pF [34] and dependent on body position and distance to ground. As an example of what this can mean in a practical setting, the value of capacitance between a series of conducting human body models and an infinite ground plane can be calculated as part of an undergraduate lab experiment. The body may be assigned a certain value of conductivity to
model the series resistance depicted on the right side of Fig. 6. Average body conductivity is 50 S/m and calculations using the modules described take about 2 minutes per model. In a lab setting, students can examine the concept of body capacitance and how this may relate to electrostatic discharge. It also provides a bevy of data for validation purposes. To date, over 300 simulations have been conducted, resulting in good correlation with experimentally obtained values.

Fig. 5. Examples of electrostatic calculations using human body models.

Fig. 6. Example experiment depicting body capacitance calculations.

**PLANE WAVE SOLUTION TO MAXWELL’S EQUATIONS**

We begin by obtaining a plane wave solution to Maxwell’s equations. In a homogeneous lossless medium with parameters \( \varepsilon = \varepsilon_0 \varepsilon_r \), \( \mu = \mu_0 \mu_r \), the electromagnetic wave propagation may be described by the wave equation for the electric field, \( E \),

\[
\frac{\partial^2 E}{\partial t^2} - c^2 \nabla^2 E = 0, \quad c = \frac{1}{\sqrt{\mu \varepsilon}}
\]  

(2a)
and by Faraday’s law of induction
\[
\frac{\partial \mathbf{H}}{\partial t} = -\nabla \times \mathbf{E}
\]  

(2b)

An electromagnetic field at single frequency \( \omega \) (a “monochromatic” field) can be represented in a phasor form, i.e.
\[
\mathbf{E}(r,t) = \text{Re}[\mathbf{E}(r)\exp(j\omega t)] = \frac{1}{2}[\mathbf{E}(r)\exp(j\omega t) + \mathbf{E}^\ast(r)\exp(-j\omega t)]
\]
\[
\mathbf{H}(r,t) = \text{Re}[\mathbf{H}(r)\exp(j\omega t)] = \frac{1}{2}[\mathbf{H}(r)\exp(j\omega t) + \mathbf{H}^\ast(r)\exp(-j\omega t)]
\]  

(3)

Substitution of this expression into Eq. (1a) yields the vector Helmholtz equation
\[
\omega^2 \mathbf{E}(r) + c^2 \nabla^2 \mathbf{E}(r) = 0, \quad c = 1/\sqrt{\mu\varepsilon}
\]  

(4)

A solution to this equation is sought in the form
\[
\mathbf{E}(r) = \mathbf{E}_i \exp(-j\mathbf{k} \cdot \mathbf{r}), \quad \mathbf{E}_i = \text{const}
\]  

(5a)

with \( \mathbf{k} \) being a constant wave vector and \( k = |\mathbf{k}| \) being the wave number. Substitution of Eq. (5a) into Eq. (4) shows that all three scalar wave equations are satisfied simultaneously if
\[
k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2} \Rightarrow k = \frac{\omega}{c},
\]  

(5b)

where \( k_x, k_y, k_z \) are three Cartesian components of the vector \( \mathbf{k} \). The resulting expression for the electric field,
\[
\mathbf{E}(r,t) = \text{Re}[\mathbf{E}_i \exp(j\omega t - j\mathbf{k} \cdot \mathbf{r})]
\]  

(6a)

is known as a plane wave propagating in the direction of vector \( \mathbf{k} \) with the speed of light. Substitution of Eq. (6a) into Eq. (2b) in the phasor form, \( j\omega \mathbf{H} = -\nabla \times \mathbf{E} \), gives the corresponding expression for the magnetic field,
\[
\mathbf{H}(r,t) = \text{Re}[\mathbf{H}_i \exp(j\omega t - j\mathbf{k} \cdot \mathbf{r})], \quad \mathbf{H}_i = \frac{1}{\eta k} \times \mathbf{E}_i
\]  

(6b)

where \( \eta = \sqrt{\mu/\varepsilon} \) is the intrinsic or wave impedance of the medium with the units of ohms. Thus, the magnetic field is always perpendicular to the electric field in a plane wave. One more important relation is obtained from Gauss’ law, \( \nabla \cdot \mathbf{E}(r) = 0 \), after substitution of the anticipated solution given by Eq. (6a). One has
\[
\mathbf{k} \cdot \mathbf{E}_i = 0
\]  

(6c)

In other words, any plane wave is characterized by the conditions
\[
\mathbf{E}_i \perp \mathbf{k}, \quad \mathbf{H}_i \perp \mathbf{k}, \quad \mathbf{E}_i \perp \mathbf{H}_i, \quad \mathbf{H}_i = \frac{1}{\eta k} \times \mathbf{E}_i, \quad \mathbf{E}_i = -\frac{k}{\eta} \times \mathbf{H}_i
\]  

(6d)

so that all the fields are constant in space in any plane perpendicular to the vector \( \mathbf{k} \). Such a plane is thus the plane of equal phase and equal amplitude; hence the name – the plane wave. Since electric and magnetic fields are both
perpendicular to the direction of propagation, the plane wave is said to be transverse electromagnetic (TEM) wave. The plane wave solution obtained in this section is often referred to as a uniform plane wave.

**ELECTRODYNAMIC MODULES AND TOPICS COVERED**

Solutions to a variety of interesting and relevant topics have been implemented in basic MATLAB using an intuitive and dynamic GUI. The user is provided maximum control of all input parameters so that adjustment of material properties, incidence angle, frequency, and other quantities can be varied to produce different solutions in real-time and explore associated effects of the input changes.

![GUI display for configuration of plane wave transmission and reflection (left) and results (right).](image)

**Plane Wave Reflection and Refraction**

The GUI, shown at left in Fig. 7, has been designed for configuring plane wave propagation. The user may adjust the angle and direction of incidence as necessary and vary wave frequency and material properties. The results, shown at right in Fig. 7, depict plane wave incidence, reflection and refraction at an interface. In this way, Snell’s laws of reflection and refraction may be explored, with identification of the incidence, reflection, and transmission angles. Power transmission, the critical angle, total reflection and the Brewster angle are additional concepts that may be explored.

**Reflection and Refraction through an Intermediate Layer**

Often, it is interesting to explore reflection and refraction of plane waves when a third layer is present, also known as the two-interface problem. The calculation of the fields inside the layered structure requires the explicit solution of the boundary conditions at the two interfaces. At the same time, it is possible to express the fields in the media of incidence and transmittance without the knowledge of the fields inside the layer (or multiple layers), using the generalized Fresnel reflection and transmission coefficients [18]. This situation is representative of a wave passing through a layer of glass or wall and helps further illustrate the concepts of transmission and signal degradation when working with lossy media. Figure 8 explores this scenario and offers insight into how transmission through such an intermediary layer can occur and how the quality of the signal is affected. Applications include transmission of an antenna or wireless device residing just below the surface of the skin and the user has complete control of the constitutive properties of all three layers.

**Zenneck Surface Waves**

The Zenneck surface wave [19]-[27] was first introduced in 1907 [19] and has had a strong influence on radio wave propagation studies over the decades. Zenneck showed that a plane interface between a lossy and lossless media could support, or guide, a surface wave. Zenneck did not show that an antenna could generate such a wave though. However, because Zenneck’s wave seemed to be a plausible explanation of radio wave propagation over large distances above lossy earth or lossy sea water (including Marconi’s transatlantic transmission), it was commonly accepted as an important contribution to antenna radiation over large distances, starting with Arnold Sommerfeld in 1911[24]. Marconi himself supported the conclusion that the electromagnetic wave was guided along the surface [25]. The idea of the Zenneck’s wave, shown at right in Fig. 8, is remarkably simple: it is possible to match a non-uniform (complex) plane wave in a lossless medium with a transmitted non-uniform wave in a lossy medium. The
interfaces between air and ground, air and sea, and air and a human body are typical interfaces for Zenneck and Zenneck-like surface waves. Figure 9 illustrates the behavior of a Zenneck wave when an additional intermediary matching layer is present. The situation is elucidates natural occurrences on an ice-covered sea [28-29].

![Fig.8. Behavior of the two-interface problem (left) and the classic Zennek wave (right).](image1)

In certain cases, a phenomenon called a surface plasmon occurs, as depicted in Fig. 9 at right. This very interesting area that is applicable to nanophotonic devices and circuits satisfies the following conditions:

i. Possesses a wavefront in air that is exactly perpendicular to the interface;

ii. Possesses a wavefront in the metal that is exactly perpendicular to the interface;

iii. Rapidly decays in air when the distance from the interface increases (is tightly bound to the surface).

### Trapped Surface Waves

Another relevant topic is that of a trapped surface wave. A dielectric sheet, which can represent a printed circuit board (PCB), situated above a perfectly conducting metal ground plane supports a trapped surface wave. In this case, the trapped surface wave is essentially a waveguide mode that extends into the surrounding space. This is depicted at left in Fig. 10, which displays a wave propagating through a channel and spilling into the media above.

![Fig.9. Zenneck wave with an intermediary layer (left) and surface plasmon (right)](image2)

**RESULTS AND CONCLUSIONS**

The modules described in this paper, which may be obtained free of charge via the Internet [35], offer students and educators with a highly interactive and dynamic set of tools that can provide a greater level of understanding when learning and teaching the concepts of electromagnetics. The modules can and have been incorporated into a standard electrical engineering curriculum, enabling a more visual and stimulating approach to these concepts. Students surveyed have expressed an increased understanding and intuitive ‘feel’ for the illustrated concepts as a direct result of having access to these modules. While this is not a completely scientific and exhaustive study, it is promising to know that these tools have made a difference as the end goal has always been to facilitate the understanding of
concepts inherent to electrodynamics. These complex and three dimensional ideas are often confusing to a student at first glance and challenging for the educator to thoroughly explain but are essential for further pursuits in electrical and biomedical engineering.

Fig. 10. Trapped surface wave (right) and plane wave reflection at an impedance boundary (left).

REFERENCES


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