Contactless Magnetic Gears – A Project Based Learning Approach to Understanding Gear Systems

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Abstract — A student who wanted to deepen her knowledge of mechanical gear systems and magnetism enrolled in a project-based learning undergraduate research course. Subjects covered included magnetic fields, force, and motion. A theoretical analysis and hands-on construction of a contactless magnetic gear system were used to obtain results. Calculations predicted that maximum gear torque occurred when the “magnetic gear teeth” were slightly displaced from their fully meshed position. The experimental, hands-on model confirmed the theoretical analysis; several working model gears were built, and their function demonstrated. An unexpected positive outcome was that her research stimulated broad discussions amongst her peers and communication of her reasoning became a personal triumph.

Index Terms—Contactless Magnetic Gears, Magnetic Fields, Mechanical Systems, Project-Based Learning

I. INTRODUCTION

Modern engineering and innovative designs in technology is built on principles of early science and mathematics. Undergraduate students can supplement their understanding of these fundamental concepts by supplementing standard course work with projects. In this case, the student was motivated to investigate the laws behind Magnetic fields, Mechanics, and Gears. She challenged herself to bridge the gap between theory and application during the fall 2017 semester. Additionally, she adapted her problem-solving skills according to the required task, and feedback from peers. The primary goal was to develop a richer interpretation and also to internalize laws of statics and mathematics necessary for a future in innovative research.

She drew from material learned in previous courses and supplemented this knowledge with unassigned readings relevant to ongoing class-work. Her interest in the efficiency of machines led to a project that combined magnetism and mechanical systems into a step-by-step analysis of Contactless Magnetic Gears. Under the guidance of an advisor, she chose to use in-school resources such as existing lab equipment and software, and readily available programs like Excel. This paper describes the learning methods employed in creating the analytical model in Excel and the physical gear system. It details the reasoning behind each procedure and how a evidence-based process can significantly improve a student's understanding of highly abstract concepts like magnetic fields in mechanical systems.

II. MECHANICAL GEARS

Mechanical gears meshing are a form of an interacting system, each gear wheel serving as a component of the system to transmit speed or torque. Placing several gears together with their teeth engaged forms a gear train. The mechanical advantage (MA) of these and other gear systems is the ratio of the input force to output force. In gear systems, the input force acts along the diameter of the gear. This line runs along the central axis of an imaginary circle that is concentric to the tips of each gear tooth; it is called the pitch circle, and the line is its pitch diameter. Mating gears must smoothly roll on the other’s pitch circle for effective transfer of force. For a gear train with input gear A and output gear B, forces act at a distance from the center of rotation that is equal their pitch diameter. The gear transmitting the input force is the driver, and the gear receiving it is the driven. Amount of force and torque transferred at the contact point – where teeth are engaged – is determined by the ratio of the radius of the pitch circle to the number of teeth for a pair of gears. This gear ratio R is translated across equations of force and motions acting on A and B; it is central to gear design. Both analyses focused on the force at the contact point.

\[
\frac{N_B}{N_A} = \frac{r_B}{r_A} = MA = R
\]  

(1)

Here, \((N_{A,B})\) is the number of teeth on gear A and B, and \((r_{A,B})\) are the radii of both pitch circles.

III. MAGNETIC FIELDS AND FORCES

The mechanism that causes the rotation in the driven gear is a non-contact magnetic force. The challenge for the project was to determine the magnitude of the distance and force that would enable rotation and transfer maximum torque using this magnetic force. The Civil Engineering course track that the student is enrolled in requires Analytical Physics II. Whereas the course focused on electromagnetics, the same basic principles could be applied to permanent magnets.

The strength of a magnet is specific to its metallic properties and dimensions. When several magnets are placed in a close area, each will contribute to the net magnetic field for these gears. The magnitude of the field at the contact point
is due to a non-uniform magnetic field, the changing forces between magnets on gear A contribute to transfer of torque at the contact point and to output gear B’s rotation. To capture the main interactions between magnets, only the north pole of each was modeled, and so the magnetic force \( F \) is inversely proportional to the distance \( r \) (Equation 2).

\[
F \propto \frac{B_1 B_2}{r^2} = \frac{B^2}{r^2}
\] (2)

The magnitude \( B \) could be squared since all 16 magnets, used on both gears, were N50 Neodymium of same dimensions, \( \frac{1}{4} \) “OD X \( \frac{1}{2} \)” and strength.

IV. ANALYTICAL MODEL

The analytical model consisted of two discs placed side by side and with magnets placed around the circumference. The radial alignment of the magnetic teeth increased the difficulty in measuring the magnetic forces at the contact point. I was able to simply the calculations using equation 2. Microsoft Excel was used to aggregate gear dimensions, input parameters, and formulas in a responsive spreadsheet. The initial set-up used equation 2 for two bar magnets (Fig. 1).

![Fig. 1. Setup for the calculation of the force between two magnetic poles using equation 2.](image)

To compile results for a two-gear system with a range of inputs, the student created a dynamic spreadsheet in Excel; it defines names and ranges that can be easily culled into a formula. The factors used in each calculation along with their units are listed below.

<table>
<thead>
<tr>
<th>Defined Names and Units</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic Strength</td>
<td>Tesla [T]</td>
</tr>
<tr>
<td>Disc Circumference</td>
<td>Meters [m]</td>
</tr>
<tr>
<td>Magnetic Count</td>
<td>–</td>
</tr>
<tr>
<td>Phase shift</td>
<td>Degrees [°]</td>
</tr>
<tr>
<td>Spacing between each gear</td>
<td>Meters [m]</td>
</tr>
<tr>
<td>Disc Radius</td>
<td>Meters [m]</td>
</tr>
<tr>
<td>Angle of rotation</td>
<td>Degrees [°]</td>
</tr>
</tbody>
</table>

With each term defined, magnet coordinates in the x,y plane were listed for driver \( D_1 \) and Driven \( D_2 \) (Fig. 2) The magnetic force between each magnet on each gear was calculated for angular distance, disc separation, disc size and magnetic teeth strength, and magnet amount. The excel setup considered a stationary position, restricting the relevant data to the magnetic field for the seven closest teeth on \( D_1 \) and their net effect on \( D_{2M1} \).

A simple line of conditional statements was used to put together the first table for distances between each magnet. An origin was indirectly defined by placing the driver and driven in the negative and positive domain, respectively. For each x point per gear, 16 different names were given, and likewise for the y point for a total of 64 unique names. This form of distinguishing each point was especially useful in locating errors in calculations and incorrect placement of tooth coordinates (Fig. 2).

![Fig. 2. Schematics for magnetic force of \( D_{1M1} \) to one magnet \( D_{2M1} \). Only north poles were considered. Distance vectors \( r \) between all driver teeth versus one magnet on driven are lines. Not all distances vectors for each driver tooth are shown. Arc length gives angular distance between magnets. All following schematic of gear measurements will follow same description.](image)

The net magnetic field was calculated using the superposition principle of summing all the forces between each magnet on each disc (Fig. 3).

![Fig. 3. Schematics for magnetic force of \( D_{1M1} \) to one magnet \( D_{2M1} \). A polar coordinate system was used for each tooth location presented an exact graphical representation of the two – gear system. Each graph could then be superimposed onto the other to ensure points were aligned and positioned around their respective axes. Points that were out of position indicated an error with the formula for that tooth position (Fig. 4).](image)
Another method used to identify errors visually, similarly took advantage of the symmetric nature of the system. Color conditional formatting was applied to each table of calculations to create a spectrum of data points per table of calculation. An intentional choice of red and blue hues emphasized points of maximum and minimum force concentration. Patches of color variations indicated erroneous calculations.

A fully debugged system provided an accurate simulation of rotational motion when all magnetic points on the driver were slightly displaced by increments of one degree. The angular distance from magnet to magnet on D₁ and D₂ were determined by dividing the circumference of each disc by the number of magnets and confirmed with the arc length formula. A full rotation spanned this distance, 0° − 22.5° (Fig. 5).

The final calculations for the forces between every magnetic tooth on both discs, their components, and resulting torque were computed in a table array consisting of 16 rows and 16 columns. Each cell measured every magnet’s relative distance, angle, force components and torque respectively. The sequence in which distance, force, and their geometric relationship dictate the dynamics of a magnetic field was demonstrated in the student’s vertical ranking system of table calculations. The resulting graph shows that maximum torque transfer occurs at an angular position of 11.25°. This corresponds to the position of maximum torque transfer for traditional non-magnetic gears in mesh (Fig. 6).

**V. HANDS-ON MODEL**

The hands-on model was designed and built identical to the theoretical version on excel. Each gear was constructed from wooden discs with holes drilled through the center to accommodate the gear shaft. Gear bearings were initially...
made from interconnecting plastic plumbing parts. 5/16” bolts acted as shafts, connecting discs to bearings and to the base. (Fig. 7) Positioned radially on the edges of each disc were the 16-1/4 “OD X 1/2” lengths N50 Neodymium “magnetic teeth” with north poles facing outwards. Feedback from students, children, and non-academics who played with the system indicated a lack of functionality due to the instability of the gears during rotation. Subsequently, 3/4” plywood with rubber corner stands replaced the existing base. It eliminated movement of the entire structure. Skateboard bearings, placed in the plastic ones, decreased lateral motion of each gear when engaged. Bolt locks on the bearings restrained the rotating gears, making for a smoother and better functioning demo as seen in figure 9.

Data of force versus angle was collected using two different sensors connected to a school provided laptop equipped with force analysis software (Fig. 8a). The force sensor was attached to a reference position, the contact point and a rotary motion sensor was anchored to the axis of the driver gear. To prevent rotation in the driven gear, the bolts previously used to hold the shaft in place were tightened (Fig. 8b). The force sensor was then slowly guided till the driver had gone through a rotation of about 30°. Each trial was carefully recorded and notes taken to reduce possible sources of error due to varying starting position of the force sensor. The graph obtained from the theoretical analysis was also used to determine which how to measure the experimental force. The workstation setup by the student can be seen in figure 8c.

The resulting graph was compared to the theoretical graph. Both confirmed the angular displacement of each tooth at contact point for maximum torque transfer. The set-up for the experimental analysis was a collaborative effort between the student, her advisor, and the lab technician. The student communicated her needs and used their knowledge and experience as resources. (Fig. 9)
Fig. 8. Setup for gathering experimental data. A) School provided laptop with lab software. B) Two-gear system with rotatory motion sensor attached to central axis of driver gear. C) (Below) Student’s laptop with Excel spreadsheet calculations and notebook to record each trial.

Fig 9. Graphs of theoretical analysis (orange) result with experimental (blue) result superimposed. Similar peaks and troughs in both graphs indicate that theoretical analysis approximately predicted the correct angular position of maximum torque transfer, \( \approx 11.25^\circ \).
VI. DISCUSSION

Constant and careful attention to each detail of the analyses and experiments saw that both models matched in theory and experiment. It reflected the student’s level of critical-thinking and understanding at every phase of the analysis. Discussions and comments led to an improvement in the support structure of the two-gear system. She successfully used this critique to solve the stability problems of the model without comprising the accuracy of the data. The experimental portion of the research project would have been otherwise labor-intensive and frustrating. Additionally, the detailed graph improved on results obtained in the Excel calculations.

The project exposed the student to the step-by-step research process used in engineering. She learned that to investigate, verify, and demonstrate the fundamental concepts behind contactless magnetic gears, she needed to narrow her focus to a small part of the overall mechanism. She initially felt limited in her understanding of gear trains, but after seeing how closely the analytical and graphical data matched, she felt fulfilled in having verified a small but critical mechanism of power transfer in contactless magnetic gears.

VII. CONCLUSION

Speaking with students and weekly project meetings with an advisor emphasized the need for quality data results and a concise presentation. The key-learning objective became effective communication about the importance of theory in real-world applications like Contactless Magnetic Gears. Having been formally diagnosed with High-functioning Autism, the student had found peer-to-peer interactions and expression of ideas to be a major obstacle to a career in engineering research. The project-based Undergraduate research was encouraging, challenging and contributed significantly to her education and personal growth.

References