Root Locus PD Controller Design for a Ball and Beam System

Jonathan G. Behnke and Tooran Emami
United States Coast Guard Academy

Abstract—The ball and beam is a popular system used in control laboratories. It is fairly simple in design but is also an unstable system. These properties make it an excellent candidate to test out classical and modern control methods alike. This paper aims to evaluate the ball and beam system and use the root locus design technique to generate a control system for it. The proportional – derivative (PD) will serve as the compensator for this system. Through performing this design, students will gain a better understanding of how root locus PD design works and working with unstable systems.

Keywords—ball and beam, control system, proportional derivative

I. INTRODUCTION

The ball and beam is a system used in control laboratories around the world. Examining and working with this system can help undergraduate students learn about control algorithms and how to implement them. This system is especially important because it is unstable. Many real world processes today contain unstable systems and can be very dangerous. This paper is intended to give students exposure to the ball and beam system and experience designing different types of controllers using the root locus method. Becoming familiar with control systems is especially important to these students because of the applicability of this particular field to the United States Coast Guard. One example of real control systems being used and relied upon is the vapor control system. This system is regulated by the United States Coast Guard to keep facilities that engage in dangerous vapor transport safe. As a service, the Coast Guard is in charge of creating and enforcing the standard for this control system. This is one process that highlights how dangerous a controller failure could be. This example also shows that control systems are important to the Coast Guard in order for them to fulfill their role of keeping the public safe.

As more and more systems in daily life become automated, the public has put an increasing amount of trust into the safety of devices. One example of such important and potentially dangerous systems is the flight control of the X-29 aircraft. Stein talks about this aircraft which was designed to be aerodynamically efficient at high speeds which reduced the control stability of the plane [1]. This unstable system was a result of the center of pressure of the aircraft being ahead of its center of gravity. The instability of flying this plane manually equates to balancing a one foot long broomstick [1]. This example illustrates the obvious importance of working with unstable systems and perfecting control algorithms in the laboratory. In this case, the pilot of the aircraft relies heavily on the ability of the flight controller to perform its assigned task reliably.

Diego Colon developed the system identification of a nonlinear ball and beam control system [2]. In this paper the authors focus is on the fact that in most ball and beam studies, the ball is assumed to roll without slipping. There is no coulomb friction and/or collisions in the ends of the beams. In order to accurately identify the system more and have more appropriate controller design, this study takes those factors into consideration. In the ball and beam system, the ball travels down the beam until it collides with the walls at the end of the beam. These collisions have some elasticity which determines how the velocity of the ball is changed after impact [2]. The coefficient of restitution is a difficult parameter to measure due to the fact that it depends heavily on the exact angle of the ball’s impact with the wall. Experimental tests are used to determine the coefficient of restitution and elasticity of the bands, which control the movement of the beam. Using these values, they proposed an H-infinity controller design for the system. A design of this nature gave them good disturbance rejection properties, good tracking of reference signals, and rejection of noise. They developed an H-infinity control system based on the model that they came up with and were able to reduce the sensitivity of the plant to uncertainties.

In another study, Tricha Anjali looks at the optimal control for a ball and beam system [3]. In this study, the authors propose a proportional-integral-derivative (PID) controller for their plant. In this paper the importance of the ball and beam system is emphasized and why, because it is an unstable system, it is extremely valuable in testing of control algorithms. Anjali mentions that specifically in the ball and beam system, the problem of balancing the ball at a certain point on the beam is similar to controlling the pitch of an aircraft [3]. This paper focuses on a PID controller for the system and proposes a method for calculating control parameters known as the genetic algorithm (GA). The genetic algorithm is effective in solving complex optimization problems. This algorithm operates by generated a random initial population which is then converted into binary strings. Binary strings are then decoded into real numbers and the objective function is evaluated. Once the function is evaluated, a fitness value is assigned to each member of the initial population by the GA. Each population is then acted upon by the GA and a new population is created. This process continues until the user specified stopping criteria are met and each new iteration is termed a generation [3]. This algorithm provides a means other than root locus criteria to calculate control parameters. Anjali uses the Quanser ball and beam system and implements the control system through the Quarc platform which is the same system we will use in our testing. The paper talks about the cascade control system that makes up the ball and beam. There is one
outer loop which controls the ball position and an inner loop which controls the servo position. The inner loop controller calculates the angle required for the ball to be held at a certain place. The servo control calculates the input voltage required to bring the beam to that specific angle [3].

In Anjali’s study the genetic algorithm is run multiple times to calculate the required control parameters for the ball position and servo position controllers. These parameters were then implemented in real time to observe their performance. With the calculated values, the system is able to track the reference trajectory and perform ball position control well. The cascaded PID-PID control method was proposed in this paper and executed using parameters found using the genetic algorithm. In this study a proportional-derivative (PD) controller for the ball and beam system will be implemented. Since there are many methods of calculating control parameters this paper will focus on a root locus method of obtaining these values. There are other studies which show alternate methods such as “Obtaining an optimum PID controller for unstable systems using current search” by D. Puangdownreong which might be worth exploring. The main goal of this research is to use the root locus design method to design a PD controller and measure the performance of this compensator.

II. PROBLEM STATEMENT

A. Ball and Beam Modeling

Prior to controller design, the ball and beam system must be modeled in order to obtain an appropriate transfer function. This study will use the Quanser ball and beam system which is made up of two plants: the Rotary Servo and the Ball and Beam. The servo transfer function relates the servo load gear position $\theta_l(t)$ to the input voltage, $V_{in}(t)$. This transfer function is provided by the Quanser workbook as:

$$G_s(s) = \frac{K}{s(\tau s + 1)}$$ (1)

The parameters of $K$ and $\tau$ are given as:

$$K = 1.53 \text{ rad}/(V \cdot s)$$ (2)

$$\tau = 0.0248 \text{ s}$$ (3)

Knowing the transfer function for the servo position, the ball and beam system must be analyzed to provide the full transfer function for the entire plant. The equations presented here are based on those given in the Quanser Student Workbook [4] and more detailed explanation can be found there. The ball and beam transfer function relates the position of the ball relative to the angle of the beam. The equation of motion for the ball can be found using Newton’s Law of Motion:

$$m_b \left( \frac{d^2}{dt^2} x(t) \right) = \sum F$$ (4)

which says the sum of the forces on the ball equals the mass of the ball times the ball’s acceleration.

The forces acting on the ball can be simplified to two major components, the force from the ball’s inertia ($F_{\text{momentum}} = \frac{F_{\text{momentum}}}{r_b}$) and the translational force generated by gravity ($F_{\text{gravity}} = m_b g \sin \alpha(t)$). Substituting the equations for these two forces into equation (4) and solving for linear acceleration gives the following equation:

$$\frac{d^2}{dt^2} x(t) = \frac{m_b g \sin \alpha(t) r_b^2}{m_b r_b^2 + f_b}$$ (5)

Taking into consideration the servo dynamics will provide for equation (5) to be put in terms of the servo load shaft angle $\theta_l(t)$. The relationship between the angle of the beam and the servo load shaft angle can be derived using the sin function and simplified to the following:

$$\sin \alpha(t) = \frac{\sin \theta_i(t) r_{arm}}{L_{beam}}$$ (6)

Substituting equation (6) into equation (5) will result in the equation for ball acceleration in terms of servo shaft angle $\theta_l(t)$. Making the small angle approximation, the $\sin \theta_i(t)$ term can be simplified to $\theta_i(t)$ which will make the equation linear.

$$\frac{d^2}{dt^2} x(t) = \frac{m_b g \sin \theta_i(t) r_{arm}r_b^2}{L_{beam}(m_b r_b^2 + f_b)}$$ (7)

Taking the Laplace transform of equation (7) and substituting the parameter $K_{bb}$ for the coefficients results in the following transfer function for the ball and beam system:

$$G_{bb}(s) = \frac{X(s)}{\theta(s)} = \frac{K_{bb}}{s^2}$$ (8)
B. PD Controller Design

In order to design a controller for the ball and beam system, the servo transfer function must be evaluated in cascade with the ball and beam transfer function. As seen in figure 2, two PD controllers will be developed. One controller will be for the voltage servo angle relationship and another for the servo angle ball position relationship.

Design for these two PD controllers will be based on the root locus technique. The approach for this design is outlined in the paper by O’Brien and Watkins [5]. In the root locus technique there is a certain design point specified which is calculated from a desired percent overshoot and settling time. The two components that make up the design point ($\sigma_d$ and $\omega_d$) are calculated from percent overshoot and settling time using the following equations:

$$\sigma_d = \frac{4}{T_s}$$  \hspace{1cm} (9)

$$\omega_d = -\frac{\sigma_d \cdot \pi}{\ln \%OS}$$  \hspace{1cm} (10)

Furthermore, the angle of the compensator at the design point must satisfy the following angle criterion

$$\angle G_c(s_0) = 180^\circ - \angle G_p(s_0)$$  \hspace{1cm} (11)

Using equation (11) to calculate the desired compensator angle the compensator zero is computed using the following

$$z = \sigma_d + \frac{\omega_d}{\tan \angle G_c(s_0)}$$  \hspace{1cm} (12)

Equation (12) yields the compensator zero which means the compensator transfer function is

$$G_c(s) = s + z$$  \hspace{1cm} (13)

Using this function, the proportional ($K$) portion of the compensator can be calculated using the following magnitude criterion

$$K = \frac{1}{|G_c(s_0)G(s_0)|}$$  \hspace{1cm} (14)

The resulting $K$ and $G_c(s)$ from this approach combine to produce the PD compensator for a given system. In this particular instance, a PD controller will be developed for the servo in the inner loop of the cascade system. The inner loop will then be converted into one closed loop transfer function. This closed loop transfer function will then be combined with the ball and beam transfer function to produce one final transfer function which needs compensated. The second PD controller will be designed based on this function and this will be the outermost compensator.

III. RESULTS

Following the root locus design method described above and MATLAB, a PD compensator was designed for the servo using the transfer function provided in equation (1) and parameters given in equations (2) and (3). The PD controller that was designed is as follows:

$$PD_1 = 0.052994(s + 174)$$
This controller was placed in cascade with the servo transfer function and then the closed loop feedback function was calculated to be

\[ G_{CL1} = \frac{G_s(s)PD_1}{1 + G_s(s)PD_1} = \frac{3.2432(s + 174)}{s^2 + 43.24s + 564.2} \]

The first closed loop transfer function was placed in cascade with the ball and beam transfer function provided in equation (8) and another PD controller was designed for this combined transfer function and is as follows:

\[ PD_2 = 7.7012(s + 1.015) \]

Combining this compensator with the rest of the system resulted in the following closed loop transfer function

\[ G_{CL2} = \frac{G_{CL1}PD_2G_{bb}(s)}{1 + G_{CL1}PD_2G_{bb}(s)} = \frac{10.447(s + 1.015)(s + 174)}{(s^2 + 4s + 4.465)(s^2 + 39.24s + 413.2)} \]
IV. CONCLUSIONS

Through study of the ball and beam system, greater understanding of unstable systems and methods for designing PD controllers was obtained. The method of root locus PD design was followed in order to obtain the compensator functions on the cascade system. The result of these two controllers was an overall system percent overshoot of less than 19.8% and a settling time of 2.7 seconds. Future work would include applying the PD compensator to the real system. Additionally further tuning of the control system with PID controllers using the root locus method and compare and improve that response to the response found here.

REFERENCES